## UNIVERSITY OF CALCUTTA

Mode of Examination: Online
M.Sc. (Computer Science) Semester - III Examination, 2021

Subject: Computer Science
Paper Code \& Name: CSM303 (CBCS A) \& Theory of Computation

## Date: 21.01.2022

Time and Duration: 12:00 pm to 3:00 pm (3 hours)
Please note the following instructions carefully:
Promise not to commit any academic dishonesty.
Marks will be deducted if the same/similar answers are found in different answer scripts.
Candidates are required to answer in their own words as far as applicable.
Each page of the answer scripts should have your University Roll \# on the right-top corner.
The name of the scanned copy of the answer script will be of the following format:
(Example: CSM-303A-TOC-My Roll Number.pdf)
The subject of the mail should be the file name only.

## The scanned answer script is to be sent to CUCSE2020@ gmail.com

The report should have the top page (Page \#1) as an index page; mention page number(s) against the answer of each question number.
Extra 30 minutes is allowed for uploading the answer script.
The answer script may not be accepted after the scheduled time.
Answer Question No. 1, 2, and any Four from the rest.

1. Answer any 5 questions:
$[5 \times 2=10]$
a. Show an example that if $M$ is an NFA that recognizes language $C$, swapping the accept and non-accept states in M doesn't necessarily yield a new NFA that identifies C' ( complement of C)
b. Do NFAs recognize the class of languages closed under complement? Explain your answer.
c. What will be the result if we remove a finite set of strings from a regular language? Justify your answer.
d. Consider the set of all strings of odd length on the alphabet $\{0,1\}$. Express this set in the form of a regular expression.
e. Construct regular expression for the following language over alphabet $\{\mathrm{a}\}$
$\left\{\mathrm{a}^{\mathrm{n}} \mid \mathrm{n}\right.$ is divisible by 2 or 3 or $\left.\mathrm{n}=5\right\}$
f. How does a recursive language differ from a recursively enumerable language?
g. Give the state diagram of a DFA recognizing the following language
$\{\mathrm{w} \mid$ every odd position of w is a 1$\}$
h. Define Chomsky Normal Form and Greibach Normal Form of CFG. Give a suitable example of each.
2. Answer any 5 questions:
a. Which of the following languages are regular? Justify your answer.

$$
\begin{array}{ll}
\text { i. } & \mathrm{L} 1=\left\{\mathrm{xy}| | \mathrm{x}\left|=|\mathrm{y}|, \mathrm{x}, \mathrm{y} \in\{0,1\}^{*}\right\} .\right. \\
\text { ii. } & \mathrm{L} 2=\left\{\mathrm{xy}| | \mathrm{x}\left|=|\mathrm{y}|, \mathrm{x}, \mathrm{y} \in\{0,1\}^{*}, \text { and } \mathrm{y} \text { contains a } 1\right\} .\right.
\end{array}
$$

b. Give the regular expressions generating the following languages. In all cases, the alphabet is $\{0,1\}$
i. $\quad \mathrm{L}_{1}=\{\mathrm{w} \mid \mathrm{w}$ does not contain 100 as a substring \}
ii. $\quad L_{2}=\{\mathrm{w}$ starts with 0 and has odd length or starts with 1 and has even length \}
c. Give an informal description of the language accepted by the following NFA.

d. Give an equivalent grammar in CNF for the following CFG:

$$
\begin{aligned}
& \mathrm{S} \rightarrow \mathrm{aSbb} \mid \mathrm{T} \\
& \mathrm{~T} \rightarrow \mathrm{bTaa}|\mathrm{~S}| \epsilon
\end{aligned}
$$

e. Show that the regular languages are closed under CYCLE operation, defined as follows.
$\operatorname{CYCLE}(L)=\left\{x y \mid x, y \in\{0,1\}^{*}\right.$ such that $\left.y x \in L\right\}$
For example , if $\mathrm{L}=\{01,011\}, \operatorname{CYCLE}(\mathrm{L})=\{01,10,011,110,101\}$
f. Construct a DFA for $L=\left\{a^{n} c^{m} \mid n \geq 2, m \geq 3\right\}$
g. Let $\mathrm{G}_{1}=\left\{\mathrm{V}_{1}, \Sigma, \mathrm{~S}_{1}, \mathrm{P}_{1}\right\}$ be a right-linear grammar and $\mathrm{G}_{2}=\left\{\mathrm{V}_{2}, \Sigma, \mathrm{~S}_{2}, \mathrm{P}_{2}\right\}$ be a left-linear grammar. Assume that $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ are disjoint. Consider the linear grammar $\mathrm{G}=\left(\{\mathrm{S}\} \cup \mathrm{V}_{1} \cup \mathrm{~V}_{2}, \Sigma, \mathrm{~S}, \mathrm{P}\right)$ where S is not in $V_{1} \cup V_{2}$ and $P=\left\{S \rightarrow S_{1} \mid S_{2}\right\} \cup P_{1} \cup P_{2}$. Prove that $L(G)$ is regular.
3. a. Design a Single Tape Single Head (STSH) Turing machine to compare two unary symbol strings which are stored on the tape at positions after pointers A and B, respectively. Draw the suitable state transition diagram and mention your assumptions.
b. Prove that there exist more languages than there exist Turing Machines.
4. a. Write an algorithm to construct the regular expression from a deterministic finite automaton. Explore the algorithmic steps for the following DFA.

b. State and prove the pumping lemma for the regular set.
5. a. Construct a CFG equivalent to the following PDA.
$\operatorname{PDA}=\{(\mathrm{p}, \mathrm{q}),(0,1), \delta, \mathrm{p}, \mathrm{q},(\mathrm{Z}, \mathrm{X})\}$, where p is initial state, q is final state.
$\delta$ is defined as $\delta(\mathrm{p}, 0, \mathrm{Z})=(\mathrm{p}, \mathrm{XZ}), \delta(\mathrm{p}, 0, \mathrm{X})=(\mathrm{p}, \mathrm{XX}), \delta(\mathrm{p}, 1, \mathrm{X})=(\mathrm{q}, \mathrm{\epsilon}), \delta(\mathrm{p}, 1, \mathrm{X})=(\mathrm{p}, \epsilon), \delta(\mathrm{p}, \mathrm{\epsilon}, \mathrm{Z})=(\mathrm{p}, \mathrm{\epsilon})$
b. Construct an equivalent PDA for the following CFG

$$
\begin{aligned}
& \mathrm{S} \rightarrow \mathrm{aAB} \mid \mathrm{bBA} \\
& \mathrm{~A} \rightarrow \mathrm{bS} \mid \mathrm{a} \\
& \mathrm{~B} \rightarrow \mathrm{aS} \mid \mathrm{b}
\end{aligned}
$$

6. a. Let $\sum=\{\mathrm{a}, \mathrm{b}, \mathrm{c}\}$ and $\mathrm{L}=\left\{\alpha \mathrm{a} \alpha^{\mathrm{R}} \mathrm{a} \alpha \mid \alpha \in\{\mathrm{b}, \mathrm{c}\}^{*}\right\}$. Show that L is not context free
b. Let $L=\left\{a^{5 k+1} b^{3 k-2}\right\} \subseteq\{a, b\}^{*}$. Write CFG G with $L(G)=L$
7. a. Provide an algorithm that takes as input a CFG G, a string $x$, and decides whether $x \in \mathrm{~L}(\mathrm{G})$. Find the complexity of the algorithm.
b. Explore the algorithmic steps for the following grammar and string $x=$ ababa

- $\mathrm{S} \rightarrow \mathrm{AB} \mid \mathrm{BC}$
- $\mathrm{A} \rightarrow \mathrm{BA} \mid \mathrm{a}$
- $\mathrm{B} \rightarrow \mathrm{CC} \mid \mathrm{b}$
- $\mathrm{C} \rightarrow \mathrm{AB} \mid \mathrm{a}$

8. a. Show that the language $\mathrm{L}=\left\{v w v\left|v, w \in\{\mathrm{a}, \mathrm{b}\}^{*},|v|=2\right\}\right.$ is regular
b. Why do we need to convert a left-recursive grammar to its equivalent right-recursive form?
c. Comment on: "equivalence of PDA's and CFL's."
